On the Design of Diaphragms Capable of Continuous Tuning.[*]

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Section 1.—*Introduction.*

In calculating the acoustic characteristics of an electrically operated submarine sound-generator fitted with a spring-controlled diaphragm, it was found that the apparatus would not radiate efficiently at a given power input and frequency, unless the effective diameter was, within fairly narrow limits, of the correct size, and unless the fundamental pitch was very close to the frequency of the alternating current supplied. The calculations also showed that a non-periodic diaphragm (*i.e.*, one in which there is no elastic restoring force) could not be set into vibration at frequencies in the neighbourhood of 500 cycles without incurring very high copper losses. A similar conclusion was reached with regard to the aperiodic diaphragm. It was concluded that the best efficiency could be secured by operating the sound-generator with alternating current of the same frequency as that of the diaphragm.

Similarly, in most forms of sound-receiving apparatus, the response to submarine sound-waves is most marked when the size of the diaphragm is correctly chosen and when its fundamental pitch is in resonance with the frequency of the incident waves. In the field of submarine acoustic signalling it is evident that a great advantage is to be derived by utilising a means of continuously tuning the diaphragms of the sound-receivers or generators over a certain range of frequencies, thus realising selective transmission and reception. At the same time advantage is taken of the great increase in sensitivity characteristic of highly selective receivers, in which the damping may be reduced to a minimum in view of the possibility of being able to "tune in" to incoming waves as is now done in radiotelegraphy.

Section 2.—*On the Design of Pressure-tuned Diaphragms.*

In order to meet the need for a means of continuously altering the pitch of diaphragms forming part of submarine sound-generators or receivers, the method of construction shown in fig. 1 was adopted. The diaphragm is an accurately turned solid of revolution, preferably fashioned from a single piece of metal or other elastic substance having a low elastic hysteresis and as high an elastic limit as possible. It consists of a solid central portion, C, and of a heavy rim, R, connected by a relatively thin annulus, A. The relative dimensions of C and A may be calculated to give any frequency required in practice, as well as to provide for the maximum range of tuning possible for a material of given elastic properties. The rim of the diaphragm fits very accurately on a slightly tapered mounting of the body, B. The form of the latter may be varied to suit the conditions under which the diaphragm is to be employed. By the application of gaseous pressure (or suction) over the interior of the diaphragm (supposed to be in gas-tight connection with the body), it is caused to bulge slightly (outwards or inwards). As the solid centre is practically undeformed by its displacement relative to the rim, the distance between the outer edge of the former and the inner edge of the latter is increased (or diminished) by an amount depending on the magnitude of the pressure or suction applied. The tension of the thin annulus is thus increased or diminished and as a result the frequency of the vibrations of the central part of the diaphragm is raised or lowered. By this means the fundamental pitch of the diaphragm may be raised or lowered continuously between certain limits by an alteration of gaseous pressure.

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[*] These researches were carried out in June and July, 1917, in connection with researches on submarine acoustics carried out for the Board of Invention and Research, with whose permission the following account is now published. Further details of applications to submarine sound-generating and receiving apparatus will be found in the British Patent Specification 131,041 (1918), "Improvements in and Relating to the Tuning of Diaphragms for Generating or Receiving Sound Waves."

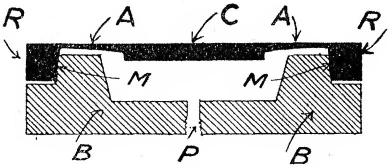

Fig. 1.—Tunable Diaphragm.

M, bronze body ; C, rigid centre ; A, annulus ; R, rigid rim of diaphragm ; M, accurately ground mounting.

The range of tuning which can be effected in this way will depend not only on the design of the diaphragm, but on the material employed in its construction and on the method of mounting. It is evident that a limit is set to the tension allowable in the annulus as soon as the stress-intensity reaches the elastic limit for the material. It is shown by an approximate mathematical investigation that this occurs initially over the inner boundary of the annulus. By suitably varying the thickness of the annulus so as to give a "diaphragm of uniform strength" the radial stress-intensity may be equalised throughout the material, and for a given pitch and elastic limit, the range of tuning may be considerably increased. It was also found that the range of tuning possible within the elastic limits of the diaphragm material varied very slowly with the ratio of radii of the annulus. This conclusion was also verified by experiment and a ratio 2 : 1 finally adopted as most convenient in practice.

Below are given predetermined characteristics of a 3-inch microphone diaphragm:

CHARACTERISTICS OF 3-INCH MICROPHONE DIAPHRAGM.

Inner radius of annulus 1·75 cm., outer radius 3·50 cm.; diaphragm shaped to uniform strength and fundamental pitch of 450; thickness of annulus at inner radius 0·038 cm., at outer radius 0·024 cm.

Mild Steel.—Young's modulus 30×10^6 lbs./sq. in.; elastic limit 50,000 lbs./sq. in.; theoretical frequency-range 450–1210.

Nickel-Chrome Steel.—Young's modulus 30×10^6 lbs./sq. in.; elastic limit 150,000 lbs./sq. in.; theoretical frequency-range 450–1980.

The above values for frequency-range are based on the supposition that the rim and solid centre are absolutely rigid and unyielding. In practice, it is to be expected that the actual limits attained will fall somewhat short of those calculated theoretically, owing to a sensible yielding at the circumference of the diaphragm. The importance of securing as closely fitting and as rigid mountings as possible is thus evident: otherwise a large bulge will be required to increase the tension in the annulus to the required degree, resulting in overstress through the effect of large bending stresses not contemplated by theory.

Section 3.—*Experimental Tests of Method of Tuning Diaphragms.*

In order to test the method of tuning diaphragms by pressure or suction, it was decided to embody the invention as part of a simple microphone receiver adapted to the needs of ordinary navigation for receiving signals from submarine sound-generators. Many of these are now established in American and Canadian waters as submarine fog-alarms, and it is probable that their use for this purpose will largely increase in the future. The pitch of these sound-generators is about 500. As it is hardly to be expected that the pitch of such generators can be maintained uniformly constant, it is evident that the receiving microphones, to be most efficient (as well as the telephone receivers used with them) should be capable of continuous tuning over a limited range in the neighbourhood of this frequency.

During a series of tests carried out in July and August, 1917, several types of diaphragms and methods of mounting were tried, with more or less successful result, in the matter of tuning. To obtain the best results it is found essential to avoid the use of rubber or cement packing, and even of soldered contacts. Finally, the type of diaphragm shown in fig. 1, embodied as part of the microphone receiver as shown in fig. 2, was found to give satisfactory results. The heavy rim of the diaphragm was carefully ground to fit the bronze mounting. It is easily seen that pressure or suction tends to bulge the central solid portion of the diaphragm inwards or outwards, thus increasing the tension in the thin annulus. As a result, the stresses produced in the rim of the diaphragm cause the former to grip the mounting more tightly as the pressure or suction is increased. If the taper of the mounting is kept sufficiently small (about 10°), the friction brought into play is sufficient to prevent the diaphragm being forced off by pressure.

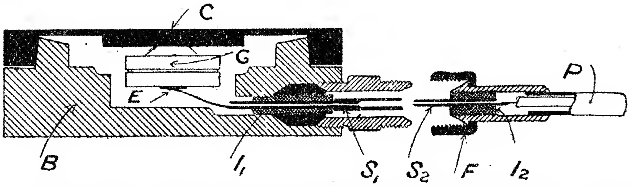

FIG. 2.—Submarine Microphone Receiver fitted with Tunable Diaphragm.

G, carbon granule microphone button ; E, platinized contact to bayonet socket S_1, S_2, mounted in insulating bushings I_1, I_2 ; F, watertight coupling connecting copper tube, P, to microphone body, B. An insulated conductor is passed through the tube and connected through S_2, S_1, E to the microphone button, G. The copper tube itself is the return circuit.

The frequency tests were carried out in a cylindrical tank (57 cm. in diameter) containing water to the depth of 1 metre. To generate sound waves in the water, an ordinary bi-pole receiver was made tight by cementing the diaphragm in position. This was connected to a source of alternating current known as the " Vreeland Oscillator." The alternating current supplied by this apparatus to a circuit free from iron-cored inductances is known to be free from harmonics, and was found especially suited to the work in hand. The frequency was varied over the range 350–1600 by inserting suitable inductances and capacities in the main circuit of the oscillator, and was measured by a specially accurate sonometer method.* By means of a telephone magnet of high resistance connected across the terminals of the apparatus, a steel wire under known tension could be adjusted in length to vibrate in resonance with the frequency of the periodic variations of magnetic flux. The wire was specially mounted to avoid frictional losses at its extremities when vibrating, so that the resonance was extremely sharply defined and determinate. By means of this precision " frequency meter," no difficulty was met with in determining frequencies over the range 350–1600 to an accuracy of one part in a thousand.

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* This precision frequency meter is described in greater detail in a paper by the writer "On the Determination of the Electrical and Acoustic Characteristics of Telephone Receivers," 'Journal of the Franklin Institute,' pp. 616–617 (May, 1919).

The microphone was immersed to a depth of 38 cm., and was connected to a set of dry batteries and to a high-resistance telephone head-piece in the usual way. By means of an air-pump connected to a steel reservoir and mercury manometer, known pressures could be applied to the microphone diaphragms through the copper tube forming one branch of the electrical circuit. The return circuit, consisting of a rubber insulated wire threaded loosely through the copper tube, was completely protected in this way from inductive effects and "strays," which are always troublesome when listening for very faint sounds.

To obtain the variation of resonance frequency with pressure, the inductances connected to the Vreeland Oscillator were adjusted to produce a note of the desired frequency in the tank. The air-pressure or suction on the diaphragm was then slowly altered until the note heard in the telephone head-piece was a maximum. The pressure giving resonance could be adjusted so that a variation of 2 mm. of mercury gave rise to a noticeable change of audibility. The curve connecting the pressure (corrected for

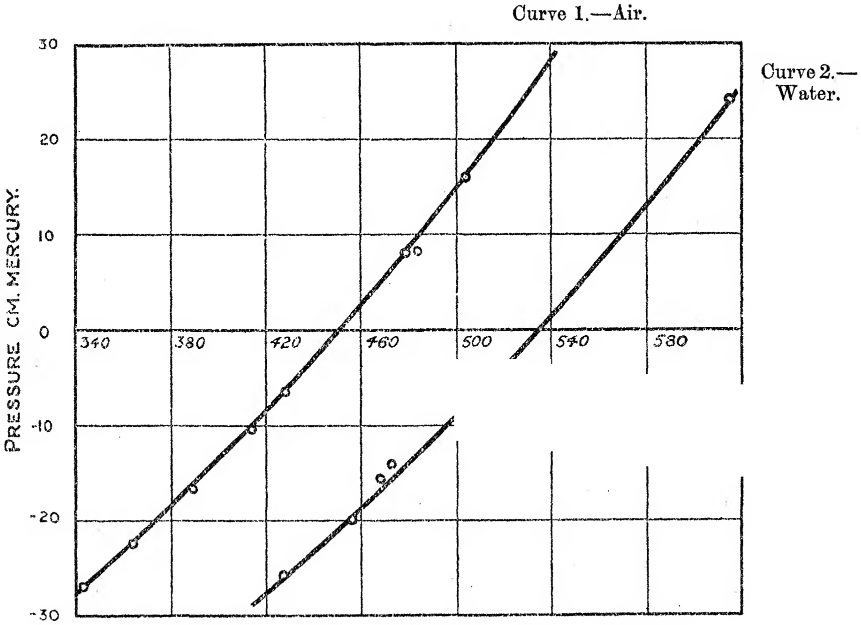

Fig. 3.—Tuning Curve.

hree-inch Tunable Diaphragm. Variation of resonance frequency with applied pressure. Curve 1, diaphragm vibrating in air; Curve 2, diaphragm vibrating in water.

depth of immersion) with resonance-frequency will be referred to as the "tuning-curve" of the diaphragm employed.

A similar curve was obtained for the diaphragm vibrating in air. In this instance the vibrations were excited by a telephone-magnet placed at a few millimetres distance from the steel diaphragm. The two sets of curves thus obtained are shown in fig. 3. It will be noticed from Curve 1 that it is possible to tune from 320 to 480 by variations of pressure within the range +10 cm. to −30 cm., which is easily furnished by the lungs alone.

It was found that the diaphragm showed signs of over-stress for pressures in the neighbourhood of ±35 cm. mercury. As the pressures approach these limits the tuning became less sharp, so that working pressures were subsequently kept well within this range. From previous tests it was ascertained that overstressing a diaphragm did not in any way impair its practical usefulness. The tuning curve was simply shifted parallel to itself, indicating that the effective elasticity of the diaphragm (at zero pressure) had been changed. The sensitiveness of the diaphragm did not appear to have suffered; in fact, diaphragms are most easily adjusted to a required tuning curve by overstressing.

Section 4.—*Note on the Experimental Determinations of the Effective Constants of Diaphragms.*

The shape of the diaphragm is designed to reduce to a minimum the part played by the excitation of overtones. Consider the diaphragm as an elastic system having a single degree of freedom. Denote by x the displacement of the solid central portion of the diaphragm from the zero position resulting from the application of pressure. In a position of equilibrium (under pressure) the elastic restoring force required to balance the applied pressure may be represented by an expression of the form

$$\mathrm{F} = \mu_0 x + \lambda x^3. \tag{1}$$

Here μ_0 is a constant depending on the elastic moduli and on the dimensions of the annulus, and is independent of the applied pressure. The term λx^3 is a third order term, in which λ depends on the applied pressure. For a small displacement, ξ, about the new position of equilibrium, the effective restoring force is

$$\delta\mathrm{F} = (\mu_0 + 3\lambda x^2)\,\delta x, \quad \text{or} \quad \delta\mathrm{F} = \mu\xi,$$

where $\mu = \mu_0 + 3\lambda x^2$ and δx is identified with ξ. We may thus write for the equation of motion representing the free vibrations

$$m\ddot{\xi} + \kappa\dot{\xi} + \mu\xi = 0. \tag{2}$$

The elastic restoring force, μ, and its variation with pressure may be approximately determined by observing a number of stress-strain curves relative to various equilibrium positions. It may also be predetermined roughly from the elastic theory of such diaphragms. The constant, κ, depends (i) on the energy absorbed at the microphone contact; (ii) on that dissipated at the diaphragm mounting, and by elastic hysteresis losses in the material of the diaphragm; and (iii) on the rate of acoustic energy radiation. The magnitude of κ cannot be predicted with certainty, but to obtain high sensitivity and sharp tuning it is evident that all the above factors, except that depending on acoustic radiation, should be reduced to a minimum. To obtain its value experimentally it would be necessary to examine the rate of damping of the free vibrations under various conditions.

The effective mass, m, includes not only the inertia of the vibrating diaphragm itself, but also the "acoustic mass" arising from the inertia of the water set into oscillatory motion by the vibrations of the diaphragm.* Considerable information relating to these constants may be obtained from a comparison of the tuning curves obtained in water and air, the acoustic mass in the latter medium being negligible. As long as the damping in water is not too great, the frequency, f_w, is given very approximately by the formula

$$2\pi f_w = \sqrt{(\mu/m_w)}, \tag{3}$$

If we compare f_w with the frequency, f_a, in air when the microphone diaphragm is subjected to the same pressure, so that μ is unaltered, we have

$$2\pi f_a = \sqrt{(\mu/m_a)}. \tag{4}$$

From (3) and (4) it follows that

$$m_w/m_a = (f_a/f_w)^2. \tag{5}$$

If m_w' denote the *acoustic mass* in water, and m_0 the effective diaphragm mass, we have (since the acoustic mass of air is negligible)

$$m_a = m_0, \qquad m_w = m_0 + m_w'. \tag{6}$$

From (5) and (6), it follows that the ratio of the acoustic to the diaphragm mass may be obtained in terms of the resonance frequency ratios corresponding to equal pressures on the diaphragm. If, in addition, μ is known experimentally, and also the resonance frequency in air, (4) and (6) show that m_0, and therefore m_w', may be calculated. For a piston diaphragm of radius a, vibrating in an infinitely extended medium, the acoustic mass is calculated to be

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* *Added Feb.* 28, 1921.—On the theoretical calculation of the acoustic mass of a circular diaphragm of *uniform* thickness filling an aperture in a plane wall which is in contact on one side with an otherwise unlimited mass of fluid, see a recent paper by Lamb, H., "On the Vibrations of an Elastic Plate in Contact with Water," 'Roy. Soc. Proc.,' A, vol. 98, pp. 205–216 (1920). Calculation shows that the theory is unfortunately not applicable to the tunable diaphragms with rigid centres discussed in the present paper.

$$m_w' = \tfrac{4}{3}a^3\rho_w, \tag{7}$$

where ρ_w is the density of the medium. From this equation it is possible to determine the "effective radius" of the diaphragm. As the contribution of the acoustic radiation to the damping constant of a piston diaphragm is known to be

$$\kappa_w' = \tfrac{1}{4}\rho_w \pi a^4 n^2/c, \tag{8}$$

where $n = 2\pi f_w$, and c is the velocity of sound in water, we are enabled in this way to separate out the damping due to acoustic radiation from that due to internal causes.

That the acoustic mass of a small 3-inch diaphragm is comparable to its actual mass, even when vibrating in a confined mass of water, is seen from the following Table, derived from the tuning curves of fig. 3.

Table I.

Pressure in cm. mercury	−20	−10	0	10	20
Resonance frequency in air, f_a	456	498	536	570	602
Resonance frequency in water, f_w...	375	415	450	487	515
Ratio, $m_w/m_a = (f_a/f_w)^2$	1·48	1·44	1·40	1·37	1·37

The numbers in the last row are seen to be in tolerable agreement, and show that in this instance the acoustic mass is as much as 40 per cent. of the diaphragm mass. The progressive variation of these numbers is attributed by the writer to the influence of the microphone contact employed. The brass mounting of one of the carbon buttons was rigidly screwed into the solid centre, and was insulated from the brass chamber containing the carbon granules and the remaining carbon button by a mica diaphragm. As a result, the steel diaphragm and brass chamber formed an elastically coupled system, with the result that the apparent mass of the former might be expected to vary with the frequency.

Section 5.—*Note on Conditions to be Realised to obtain Maximum Sensitiveness and Sharp Tuning.*

Through being able to continuously adjust the frequency of a diaphragm so as to be able to "tune in" to a note of given pitch, high sensitivity and sharp tuning are important objects to be aimed at in the design of submarine receiving apparatus. As is well known from the theory of the vibrations of damped systems, high sensitivity and sharp tuning are both obtained by keeping the damping small, *i.e.*, by reducing to a minimum the rate of dissipation of energy during the oscillations of the diaphragm. The following factors contribute to the energy losses mentioned :—

(1) Deviations in the shape of the diaphragm from that of a perfect solid of revolution. Through the action of such defects of figure, there is a tendency for the energy originally contained in the fundamental mode to be dissipated in higher unsymmetrical overtones.

(2) Deviations from a thoretically rigid mounting and from geometrically perfect boundary conditions, giving rise to an unsymmetrical and varying tension in the annulus. As in (1), there is a tendency for the energy of the fundamental to be transferred to and dissipated in inharmonic overtones. In addition, there are energy losses at the boundary contact between the diaphragm rim and the mounting.

(3) Losses due to the elastic hysteresis of the material employed in the construction of the diaphragm.

(4) Energy dissipation at the microphone contact.

The damping due to factors (1) and (2) may be minimised by very precise workmanship. Special grinding tools should be constructed to give an almost optical precision to the figure of the diaphragm. In particular, special attention should be paid to the manner in which the diaphragm is fitted into position on the mounting. Not only should the surface of contact be fine-ground into position, but all traces of an oil-film should be removed. The latter has been observed to give rise to considerable damping through dissipation arising in a thin viscous film, subjected to alternating compressions while the diaphragm is vibrating. Anything in the nature of rubber packing, cemented or soldered contacts, should be avoided for the same reason.

The losses due to elastic hysteresis in the material composing the diaphragm can only be minimised by the choice of a suitable steel or alloy; the material should at the same time possess as high an elastic limit as possible to give a wide range of tuning. The selection of a suitable material should be made the subject of a special investigation by constructing diaphragms of various metals and measuring their free damping constant by some such instrument as Prof. Dayton C. Miller's "phonodeik."*

The elimination of losses at the microphone contact can only be achieved by carrying out a special investigation.

It will be noticed that contributions to the damping arising from factors (1), (2), and (4) are not affected by the process of tuning by gaseous pressure. There is a possibility that elastic hysteresis losses during vibration may increase with the tension in the annulus to the detriment of sharp tuning at the upper limit of the frequency-range.

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* Described in 'The Science of Musical Sounds' (Macmillan and Co., New York).